

**WAEP Semester Two Examination, 2018**

**Question/Answer booklet**

**MATHEMATICS**

**METHODS**

**UNITS 1 AND 2 Section One:**

**Calculator-free**

**SOLUTIONS**

Student number: In figures

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In words \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Your name \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**Time allowed for this section**

Reading time before commencing work: five minutes

Working time: fifty minutes

**Materials required/recommended for this section**

***To be provided by the supervisor***

This Question/Answer booklet

Formula sheet

***To be provided by the candidate***

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

**Important note to candidates**

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

**METHODS UNITS 1 AND 2 2 CALCULATOR-FREE**

**Structure of this paper**

| Section | Number of questions  available | Number of  questions to be answered | Working  time  (minutes) | Marks  available | Percentage of  examination |
| --- | --- | --- | --- | --- | --- |
| Section One:  Calculator-free | 8 | 8 | 50 | 52 | 35 |
| Section Two:  Calculator-assumed | 13 | 13 | 100 | 98 | 65 |
|  |  |  |  | **Total** | 100 |

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**Instructions to candidates**

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1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this

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examination implies that you agree to abide by these rules.

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2. Write your answers in this Question/Answer booklet.

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3. You must be careful to confine your response to the specific question asked and to follow

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any instructions that are specified to a particular question.

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4. Supplementary pages for the use of planning/continuing your answer to a question

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have been provided at the end of this Question/Answer booklet. If you use these pages

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to continue an answer, indicate at the original answer where the answer is continued,

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i.e. give the page number.

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5. Show all your working clearly. Your working should be in sufficient detail to allow your E

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answers to be checked readily and for marks to be awarded for reasoning. Incorrect

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answers given without supporting reasoning cannot be allocated any marks. For any

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question or part question worth more than two marks, valid working or justification is T

required to receive full marks. If you repeat any question, ensure that you cancel the

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answer you do not wish to have marked.

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6. It is recommended that you do not use pencil, except in diagrams.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

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**CALCULATOR-FREE 3 METHODS UNITS 1 AND 2**

**Section One: Calculator-free 35% (52 Marks)**

This section has **eight (8)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 50 minutes.

**Question 1 (4 marks)** (a) Expand (2�� + 1)3. (2 marks)

| **Solution** |
| --- |
| (2�� + 1)3 = (1)(2��)3(1)0 + (3)(2��)2(1)1 + (3)(2��)1(1)2 + (1)(2��)0(1)3 = 8��3 + 12��2 + 6�� + 1 |
| **Specific behaviours** |
| ✓ correct method  ✓ correct expansion |

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(b) Determine the gradient of the curve �� = (2�� + 1)3 at the point (1, 27). (2 marks)

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| **Solution** |
| --- |
| ����  ���� = 24��2 + 24�� + 6  �� = 1 ⇒����  ���� = 24 + 24 + 6 = 54 |
| **Specific behaviours** |
| ✓ differentiates expression from (a) ✓ evaluates gradient |

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**METHODS UNITS 1 AND 2 4 CALCULATOR-FREE**

**Question 2 (6 marks)** (a) Evaluate ��2

��0.5when �� = 6 × 102 and �� = 9 × 104, writing your answer without the use of

scientific notation. (3 marks)

| **Solution** |
| --- |
| ��0.5 =62 × 104  ��2  √9 × 102  =363× 102  = 1 200 |
| **Specific behaviours** |
| ✓ simplifies ��2  ✓ simplifies ��0.5  ✓ correct value |

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(b) Determine the value of �� when 9�� = 27√3. (3 marks)

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| **Solution** |
| --- |
| 32�� = 33 × 312  = 372  2�� =72⇒ �� =74 |
| **Specific behaviours** |
| ✓ LHS as power of 3  ✓ RHS as power of 3  ✓ equates indices and solves |

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**See next page** SN078-122-2

**CALCULATOR-FREE 5 METHODS UNITS 1 AND 2**

**Question 3 (7 marks)** Solve each equation below for ��.

(a) 3��

�� − 5=23. (2 marks)

| **Solution** |
| --- |
| 9�� = 2�� − 10  7�� = −10  �� = −107 |
| **Specific behaviours** |
| ✓ cross multiplies  ✓ correct solution |

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(b) (�� + 3)(�� − 3) = 8��. (3 marks)

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| **Solution** |
| --- |
| ��2 − 9 = 8��  ��2 − 8�� − 9 = 0  (�� + 1)(�� − 9) = 0  �� = −1, �� = 9 |
| **Specific behaviours** |
| ✓ expands and equates to zero  ✓ factorises  ✓ correct solutions |

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(c) √2 sin �� + 1 = 0, 0° ≤ �� ≤ 360°. (2 marks)

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| **Solution** |
| --- |
| sin �� = −1√2  �� = 225°, �� = 315° |
| **Specific behaviours** |
| ✓ one correct solution  ✓ both correct solutions |

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**METHODS UNITS 1 AND 2 6 CALCULATOR-FREE**

**Question 4 (7 marks)** (a) Simplify

(i) ������(10 − 3�� + 4��2). (1 mark)

| **Solution** |
| --- |
| −3 + 8�� |
| **Specific behaviours** |
| ✓ correct derivative |

(ii) lim ℎ→0

(�� + ℎ)2 − ��2

ℎ. (1 mark)

| **Solution** |
| --- |
| 2�� |
| **Specific behaviours** |
| ✓ correct derivative |

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(b) Determine the equation of the tangent to the curve �� = ��3 − 9�� + 15 when �� = 2.

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(3 marks)

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| **Solution** |
| --- |
| ����  ���� = 3��2 − 9  �� = 2, �� = 8 − 18 + 15 = 5,����  ���� = 12 − 9 = 3  �� − 5 = 3(�� − 2) ⇒ �� = 3�� − 1 |
| **Specific behaviours** |
| ✓ correct derivative  ✓ calculates ��-coordinate and gradient  ✓ correct equation of tangent, in any form |

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(c) Determine ��(��) given ��′(��) = 6�� − 2 and ��(−1) = 6. (2 marks)

| **Solution** |
| --- |
| ��(��) = 3��2 − 2�� + ��  3(−1)2 − 2(−1) + �� = 6 ⇒ �� = 1 ��(��) = 3��2 − 2�� + 1 |
| **Specific behaviours** |
| ✓ correct antiderivative with constant ✓ correct ��(��) |

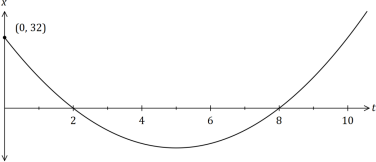
**See next page** SN078-122-2

**CALCULATOR-FREE 7 METHODS UNITS 1 AND 2**

**Question 5 (6 marks)**

A small body moves in a straight line so that its displacement �� from a fixed point �� after �� seconds is given by �� = ����2 + ���� + �� metres.

The position-time graph of the body is shown below.

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(a) Determine the values of the constants ��, �� and ��. (3 marks)

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| **Solution** |
| --- |
| �� = ��(�� − 2)(�� − 8)  32 = ��(−2)(−8) ⇒ �� = 2  �� = 2(��2 − 10�� + 16)  = 2��2 − 20�� + 32  �� = 2, �� = −20, �� = 32 |
| **Specific behaviours** |
| ✓ writes equation using roots  ✓ uses ��-intercept to find ��  ✓ expands and states three values |

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(b) Determine the displacement of the body when its velocity is 24 ms-1. (3 marks)

| **Solution** |
| --- |
| �� = 4�� − 20  4�� − 20 = 24 ⇒ �� = 11  ��(11) = 2(11 − 2)(11 − 8) = 54 m |
| **Specific behaviours** |
| ✓ equation for velocity  ✓ solves for time  ✓ substitutes for displacement |

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**METHODS UNITS 1 AND 2 8 CALCULATOR-FREE**

**Question 6 (6 marks)** The derivative of a cubic polynomial is given by ����

����= 3��2 + 6�� − 10.

The cubic passes through the point (1, −30).

(a) Determine the equation of the cubic. (2 marks)

| **Solution** |
| --- |
| �� = ��3 + 3��2 − 10�� + ��  −30 = 1 + 3 − 10 + �� ⇒ �� = −24  �� = ��3 + 3��2 − 10�� − 24 |
| **Specific behaviours** |
| ✓ antidifferentiates correctly  ✓ determines constant |

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(b) Show that the cubic has a root when �� = 3. (1 mark)

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| **Solution** |
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| �� = 3, �� = 27 + 27 − 30 − 24 = 54 − 54 = 0 |
| **Specific behaviours** |
| ✓ substitutes and obtains zero |

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(c) Determine the coordinates of the other two roots of the cubic. (3 marks)

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| **Solution** |
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| ��3 + 3��2 − 10�� − 24 = (�� − 3)(��2 + ���� + 8) = (�� − 3)(��2 + 6�� + 8)  = (�� − 3)(�� + 4)(�� + 2)  Other roots at (−2, 0) and (−4, 0). |
| **Specific behaviours** |
| ✓ obtains quadratic factor by inspection ✓ factorises quadratic  ✓ states both roots as coordinates |

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**See next page** SN078-122-2

**CALCULATOR-FREE 9 METHODS UNITS 1 AND 2**

**Question 7 (8 marks)** The first three terms, in order, of a sequence are 4�� + 3, 2�� − 1 and �� − 8.

Determine the fourth term of the sequence if

(a) the sequence is arithmetic. (4 marks)

| **Solution** |
| --- |
| �� = (2�� − 1) − (4�� + 3) = (�� − 8) − (2�� − 1) −2�� − 4 = −�� − 7 ⇒ �� = 3  �� = (6 − 1) − (12 + 3) = −10  ��4 = (3 − 8) − 10 = −15 |
| **Specific behaviours** |
| ✓ equates differences  ✓ solves for ��  ✓ states ��  ✓ correct ��4 |

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(b) the sequence is geometric. (4 marks)

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| **Solution** |
| --- |
| �� =2�� − 1  4�� + 3=�� − 8  2�� − 1  (2�� − 1)(2�� − 1) = (4�� + 3)(�� − 8)  4��2 − 4�� + 1 = 4��2 − 29�� − 24  25�� = −25  �� = −1  �� =−2 − 1  −4 + 3= 3  ��4 = (−1 − 8) × 3 = −27 |
| **Specific behaviours** |
| ✓ equates ratios  ✓ solves for ��  ✓ states ��  ✓ correct ��4 |

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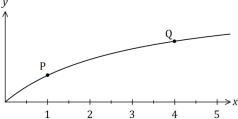
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**METHODS UNITS 1 AND 2 10 CALCULATOR-FREE**

**Question 8 (8 marks)** Let ��(��) =��

�� + 3. The graph of �� = ��(��) is shown below.



(a) Points �� and �� lie on the curve with ��-coordinates 1 and 4 respectively.

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(i) Determine ��(1) and ��(4). (1 mark)

| **Solution** |
| --- |
| ��(1) =14, ��(4) =47 |
| **Specific behaviours** |
| ✓ both values correct |

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(ii) Determine the gradient of the straight line through �� and ��. (2 marks)

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| **Solution** |
| --- |
| �� = (47−14) ÷ 3 = (16 − 7  28 ) ÷ 3 =328 |
| **Specific behaviours** |
| ✓ substitutes correctly into gradient formula ✓ correct value |

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��(�� + ℎ) − ��(��)

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(b) Use the formula ��′(��) = lim ℎ→0

ℎto determine the gradient of the curve at ��.

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(5 marks)

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| **Solution** |
| --- |
| ��(1 + ℎ) − ��(1)  ��′(1) = lim  ℎ  ℎ→0  ℎ→0(1 + ℎ  4 + ℎ−14) ÷ ℎ  = lim  ℎ→0(4 + 4ℎ − 4 − ℎ  = lim  4(4 + ℎ)) ÷ ℎ  3  = lim  16 + 4ℎ  ℎ→0  =316 |
| **Specific behaviours** |
| ✓ indicates that ��′(1) is required  ✓ substitutes function into quotient  ✓ correctly combines difference of fractions ✓ eliminates division by ℎ  ✓ evaluates limit |

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**End of questions** SN078-122-2

**CALCULATOR-FREE 11 METHODS UNITS 1 AND 2**

Supplementary page

Question number: \_\_\_\_\_\_\_\_\_

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